# Prohibiting State Aid in an Integrated Market:

## Cournot and Bertrand Oligopolies with Differentiated Products*

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**Abstract**

The prohibition of state aid in an integrated market such as the European Community is analysed in a model where firms produce differentiated products and market structure is either Cournot or Bertrand oligopoly. State aid is financed by distortionary taxation so the opportunity cost of government revenue exceeds unity. Under both Cournot and Bertrand oligopoly, if products are sufficiently close substitutes then there exists a range of values for opportunity cost where governments give state aid and where the prohibition of state aid will increase aggregate welfare. With sufficiently differentiated products, the prohibition of state aid will reduce aggregate welfare.

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## 1. Introduction

At the recent European Council meeting in Cardiff, the member states of the European Community emphasised the need to promote competition and to reduce distortions such as state aid. Although Article 87 of the EC Treaty prohibits ‘any aid granted by a member state or through state resources in any form whatsoever which distorts or threatens to distort competition’, Article 88 lists a number of exceptions that are either always exempt or may be exempted by the European Commission.[1] Since the Commission raises no objection to proposals for state aid from member states in about 80-90% of cases without any formal investigation and only 1-5% of decisions are actually negative, it is not surprising that surveys of state aid by the Commission find that the level of state aid is unacceptably high. This high level of state aid can be seen from table one, which presents some statistics on the level of state aid in the European Community over time and in the individual member states. The prohibition of state aid may be further weakened by the proposed block exemption of some forms of horizontal state aid (subsidies for SMEs, R&D, environmental protection, employment and training, and regional aid), although the aim is to allow the Commission to concentrate upon the most important cases.[2]

Despite the significance of state aid as a policy issue, there has been little economic analysis of the rationale for the European Community’s prohibition of state aid apart from Besley and Seabright (1999), Collie (2000) and the articles in Commission of the European Communities (1999c). It might be thought that the strategic trade policy literature started by Brander and Spencer (1985) would provide a rationale for the prohibition of state aid, and this is certainly the view of Besley and Seabright (1999). They argue that this literature is ‘most supportive of the EU’s stand on state aid’ and that in this literature ‘countries compete with each other in a negative-sum game of individually rational, but collectively wasteful subsidies to industry, spurred by the prospect of poaching each other’s profits in imperfectly competitive markets'. However, it is misleading to apply the strategic trade policy literature to the issue of state aid without any adaptation of the model. In Brander and Spencer (1985), an export subsidy war between two exporting countries that do not consume the exported product leads to a prisoner's dilemma type of outcome.[3] Whereas state aid generally involves domestic subsidies and the countries involved are usually both producers and consumers of the subsidised products; hence, a subsidy war will reduce the oligopolistic distortion thereby possibly increasing the welfare of all countries.

In a symmetric setting, Collie (2000) analyses the prohibition of state aid in a homogeneous product Cournot oligopoly model where a production subsidy is used as a proxy for state aid. When it is feasible to finance the subsidy with lump-sum taxes, the usual assumption in the strategic trade policy literature, he shows that a subsidy war will actually yield a Pareto-efficient outcome where price is equal to marginal cost. Obviously, in this case, the multilateral prohibition of state aid will not be beneficial. However, the outcome becomes a prisoner's dilemma when subsidies are financed by distortionary taxation and the opportunity cost of government revenue exceeds unity as in Neary (1994).[4] Collie (2000) shows that there always exists a range of values for opportunity cost of government revenue where the Nash equilibrium subsidy is positive and where the multilateral prohibition of subsidies would increase the welfare of all countries. Furthermore, he shows that this range of values includes plausible estimates of the opportunity cost of government revenue.

Since results in the strategic trade policy literature are renowned for their lack of robustness, investigating the robustness of the results in Collie (2000) would seem to be a worthwhile endeavour. Two possible sources of non-robustness spring to mind: Firstly, introducing product differentiation would lessen the effect that one country's subsidy will have on the firms in other countries. Hence, the case for the multilateral prohibition of subsidies might be weakened when products are sufficiently differentiated. Secondly, given the well-known result of Eaton and Grossman (1986), changing from Cournot oligopoly to Bertrand oligopoly might be expected to alter the results in a significant way, for example, the Nash equilibrium subsidy may not be positive under Bertrand oligopoly.

To address these issues the multilateral prohibition of state aid is analysed in a model where firms produce differentiated products and market structure is either Cournot or Bertrand oligopoly. As in Collie (2000) the model is symmetric with identical firms and identical countries, which is a reasonable assumption given that the concern of the paper is with the aggregate welfare effects of the multilateral prohibition of state aid. Adding asymmetry would mean that the gains and losses would not be equally distributed between the countries, but would not change the aggregate welfare effects in any significant way. For simplicity, demand functions are assumed to be linear and this seems reasonable given that the results in Collie (2000) held for general demand functions.

Even though state aid covers all types of subsidies granted to firms, a production subsidy will be used as a proxy for state aid in this paper as in Collie (2000). From the point of view of economic efficiency, a production subsidy is probably the most benevolent interpretation of state aid as it expands the output of firms thereby reducing the oligopolistic distortion. Thus, the results of this paper will be biased against the prohibition of state aid so if prohibition is found to be beneficial then this will be a ‘strong’ result. It would imply that other less benevolent forms of state aid, such as subsidies to fixed costs that merely keep inefficient firms in business, should definitely be prohibited.

The layout of this paper is as follows: Section two describes the basic model and derives the Cournot equilibrium outputs and prices. The Nash equilibrium in subsidies under Cournot oligopoly is obtained in section three while the multilateral prohibition of subsidies under Cournot oligopoly is analysed in section four. Section five derives the Bertrand equilibrium prices and outputs. The Nash equilibrium in subsidies under Bertrand oligopoly is obtained in section six while the multilateral prohibition of subsidies under Bertrand oligopoly is analysed in section seven. Section eight provides some concluding comments including a discussion of the policy implications of the results.

## 2. The Model

Consider an integrated market, such as the Single Market of the European Community, consisting of *M* identical countries with no barriers to trade or arbitrage between countries.[5] In each country, there is single oligopolistic firm representing the manufacturing sector that produces a nationally differentiated product for sale in the integrated market.[6] All firms are assumed to be symmetric with identical and constant marginal cost *c*. The *i*th firm receives a production subsidy $s_i$ per unit of output and produces output $x_i$ for sale in the integrated market at price $p_i$. Initially, the market structure is assumed to be Cournot oligopoly, where the strategic variables are quantities, but later in the paper it will be assumed to be Bertrand oligopoly, where the strategic variables are prices. All countries are the same size and all consumers have identical preferences with all the nationally differentiated products entering the utility function in a symmetric manner. Hence, demand is identical in all countries and demand functions for all the nationally differentiated products are symmetric. For simplicity, there is assumed to be a representative consumer in each country with quasi-linear preferences that are given by the quadratic utility function:

$$U(\mathbf{y}) = \alpha \sum_{i=1}^{M} y_i - \frac{1}{2}\left( \beta \sum_{i=1}^{M} y_i^2 + 2\gamma \sum_{j \neq i} y_i y_j \right) \tag{1}$$

where $y_i$ is consumption of the oligopolistic good produced by the firm in the *i*th country.[7] The coefficients are all positive with $\alpha > c$, and the products are assumed to be imperfect substitutes so $0 < \gamma < \beta$ where $\gamma / \beta$ is a measure of the degree of substitutability between the differentiated products that ranges from zero to one, when the products are perfect substitutes. It is straightforward to show that this utility function yields the following inverse and ordinary demand functions:

$$\begin{aligned} y_i &= \frac{1}{\Phi}\left[\alpha\left(\beta-\gamma\right)-bp_i+\gamma\sum_{j\neq i}p_j\right] \\ p_i &= \alpha-\beta y_i-\gamma\sum_{j\neq i}y_j \end{aligned} \tag{2}$$

where $b \equiv \beta+\left(M-2\right)\gamma > 0$ and $\Phi \equiv \left(\beta-\gamma\right)\left(\beta+\left(M-1\right)\gamma\right) \geq 0$. Obviously, given the quadratic utility function, these demand functions are linear in prices and output respectively.[8] Since preferences are quasi-linear, consumer surplus is a valid measure of consumer welfare that is defined as:

$$V\left(\mathbf{y}\right)=\left(U\left(\mathbf{y}\right)-\sum_{j=1}^{M}p_jy_j-z\right) \quad \text{and} \quad dV=-\sum_{j=1}^{M}y_jdp_j \tag{3}$$

In an integrated market, with perfect arbitrage and no trade barriers, the consumer price will be identical in all the $M$ member states; thus, symmetry implies that demand for the product will be identical in all member states so demand for the product of the $i$th firm is $x_i = My_i$. Hence, the aggregate ordinary demand function for the $i$th good in the integrated market is:

$$x_i = \frac{M}{\Phi}\left[\alpha\left(\beta-\gamma\right)-bp_i+\gamma\sum_{j\neq i}p_j\right] \tag{4}$$

Inverting yields the aggregate inverse demand function for the $i$th good in the integrated market:

$$p_i = \alpha - \frac{\beta}{M} x_i - \frac{\gamma}{M} \sum_{j \neq i} x_j \qquad (5)$$

In game-theoretic terms, the situation being modelled can best be described as a two stage game where the national governments set their production subsidies to maximise their national welfare at the first stage, and then the firms compete as Cournot or Bertrand oligopolists at the second stage. The case when the firms compete as Cournot oligopolists will be analysed first. As usual, the game is solved by backwards induction to obtain the subgame perfect equilibrium, and the first step is to solve the second stage of the game where the firms compete as Cournot oligopolists given the production subsidies set by the national governments. The profits of the firm in the *i*th country are:

$$\pi_i = \left( p_i - c + s_i \right) x_i \qquad (6)$$

At a Cournot equilibrium, the $M$ firms independently and simultaneously set their outputs to maximise their profits given the subsidies set by the national governments. With linear demand functions, the Cournot equilibrium will be unique and the second-order conditions for profit maximisation are bound to be satisfied since the profit functions are concave in output. Linear demand functions also imply that the outputs of the firms will be strategic substitutes so the reaction functions of the firms will be downward sloping. Assuming an interior solution, where all firms produce positive quantities, the first-order conditions for the unique Cournot equilibrium are:

$$\frac{\partial \pi_i}{\partial x_i} = p_i + x_i \frac{\partial p_i}{\partial x_i} - c + s_i = 0 \qquad i = 1, \ldots, M \qquad (7)$$

Using (5) and noting that $\partial p_i / \partial x_i = -\beta / M$ yields the system of $M$ simultaneous equations:

$$2\beta x_i + \gamma \sum_{j \neq i} x_j = M\left(\alpha - c + s_i\right) \qquad i = 1, \ldots, M \tag{8}$$

Then, solving these simultaneous equations yields the Cournot equilibrium outputs as functions of the subsidies set by the national governments:

$$x_i^C = \frac{M}{\Delta}\left[(2\beta - \gamma)(\alpha - c) + \left(2\beta + (M-2)\gamma\right)s_i - \gamma \sum_{j \neq i} s_j\right] \tag{9}$$

where $\Delta \equiv (2\beta - \gamma)\left(2\beta + (M-1)\gamma\right) > 0$ and the superscript $C$ is used to denote the Cournot equilibrium. The Cournot equilibrium output of a firm is increasing in the domestic subsidy and decreasing in foreign subsidies. Substituting the Cournot equilibrium outputs into the inverse demand functions (5) yields the Cournot equilibrium prices as functions of the subsidies:

$$p_i^C = \frac{\alpha\beta + \left(\beta + (M-1)\gamma\right)c}{2\beta + (M-1)\gamma} - \frac{(2\beta - \gamma)\beta + (M-1)(\beta - \gamma)\gamma}{\Delta} s_i - \frac{\beta\gamma}{\Delta} \sum_{j \neq i} s_j \tag{10}$$

All subsidies have the effect of reducing the price of the firm’s output but the subsidy given by the firm’s own government has a larger effect than a foreign subsidy. In this symmetric model when all governments give the same subsidy, $s_i = s$, then all firms will produce the same quantity of output and receive the same price. These symmetric Cournot equilibrium output and price are:

$$x^C = M \frac{\alpha - c + s}{2\beta + (M-1)\gamma} \qquad p^C = \frac{\alpha\beta + \left(\beta + (M-1)\gamma\right)(c - s)}{2\beta + (M-1)\gamma} \tag{11}$$

An increase in this common subsidy will increase the Cournot equilibrium output and reduce the price received by the firms. These results will be used later in the paper to analyse the effects of uniform reductions in subsidies.

## 3. Nash Equilibrium Subsidies under Cournot Oligopoly

Having derived the Cournot equilibrium of the second stage of the game, the next step is to solve the first stage of the game to obtain the Nash equilibrium in subsidies. Despite the fact that all the countries are members of the customs union, it seems reasonable to assume that each country maximises its own national welfare and attaches no weight to the welfare of the other countries in the customs union. Hence, ignoring distributional considerations, the welfare of each country is given by its consumer surplus plus the profits of the domestic firm less the cost of the production subsidy. Since the government revenue to pay the subsidy will usually be raised by some form of distortionary taxation, the opportunity cost of government revenue will be allowed to exceed unity as in Neary (1994) and Collie (2000). Hence, the cost of the production subsidy should include the deadweight loss imposed by the distortionary taxation used to finance the subsidy. Thus, the welfare of the *i*th country can be written as:

$$W_i = V(\mathbf{y}) + \pi_i - \lambda s_i x_i = V(\mathbf{y}) + (p_i - c)x_i - (\lambda - 1)s_i x_i \tag{12}$$

where $\lambda \geq 1$ is the opportunity cost of government revenue and the term $(\lambda - 1)s_i x_i$ is the deadweight loss from the distortionary taxation, which will be zero if lump-sum taxes are feasible, $\lambda = 1$.

At the first-stage of the game, the national governments each independently and simultaneously set their production subsidies to maximise their national welfare realising that the firms will compete as Cournot oligopolists in the second stage of the game with outputs and prices given by (9) and (10). Thus, the first-order conditions for the Nash equilibrium in subsidies are:

$$\frac{\partial W_i}{\partial s_i} = -\sum_{j=1}^{M} y_j \frac{\partial p_j}{\partial s_i} + x_i \frac{\partial p_i}{\partial s_i} + (p_i - c)\frac{\partial x_i}{\partial s_i} - (\lambda - 1)\left( s_i \frac{\partial x_i}{\partial s_i} + x_i \right) = 0 \qquad i = 1, \ldots, M \tag{13}$$

The first term shows that the subsidy reduces the prices of all the products bought by the consumer thereby increasing consumer surplus. The second term shows that the subsidy reduces the price received by the domestic firm thereby reducing its profits. Together these two terms represent the terms of trade effect of the subsidy. The third term shows that the subsidy increases the output of the domestic firm thereby increasing its profits if price exceeds marginal cost. This is the profit-shifting effect that is well known from Brander and Spencer (1985) and the subsequent literature on strategic trade policy. The fourth term shows that the subsidy increases the revenue that has to be raised by the government and thereby increases the deadweight loss from distortionary taxation. This will be called the distortionary taxation effect.

Since the model is symmetric, there will obviously be a symmetric Nash equilibrium where all national governments give identical subsidies, $s_i = s_N^C$, while all firms will produce identical outputs, $x_i = x_N^C$, and receive the same price, $p_i = p_N^C$, so consumption of all products will be identical in each country, $y_N^C = x_N^C / M$. Therefore, although there will be intra-industry trade, all countries will have balanced trade in the oligopolistic product. However, unlike the case of homogeneous products considered by Collie (2000), the terms of trade effect will not be zero in this case. With differentiated products, the subsidy given by a national government has a larger effect on the price of the domestic firm's product than on the prices of the foreign firms' products. Hence, although trade is balanced, the decrease in the price of the domestic firm's exports outweighs the decrease in the price of imports from foreign firms so the overall terms of trade effect will be negative.

Imposing symmetry and using (9) and (10) to evaluate the derivatives in (13) yields an implicit solution for the Nash equilibrium subsidy:

$$s_N^C = \frac{\left(2\beta+(M-2)\gamma\right)\beta+(M-1)\left(\beta+(M-1)\gamma\right)\gamma-(\lambda-1)M\Delta}{\lambda M^2\left(2\beta+(M-2)\gamma\right)} x_N^C \tag{14}$$

The Nash equilibrium subsidy is unambiguously positive when lump-sum taxes are feasible, $\lambda = 1$, and by differentiating (14) it can be shown to be decreasing in the opportunity cost of government revenue:

$$\frac{\partial s_N^C}{\partial \lambda} = \frac{-\left(2\beta+(M-1)\gamma\right)\left(x_N^C\right)^2}{\lambda^2 M(\alpha-c)\left(2\beta+(M-2)\gamma\right)}\left[\beta(2\beta-\gamma)+(2M\beta-\gamma)\left(2\beta+(M-1)\gamma\right)\right]<0 \tag{15}$$

Obviously, an increase in the opportunity cost of government revenue will reduce the governments' incentives to use subsidies thereby leading to a reduction in the Nash equilibrium subsidies. Hence, with distortionary taxation, the Nash equilibrium subsidy will only be positive if the opportunity cost of government revenue is less than the critical value $\lambda_S^C$, which is obtained by setting the numerator of (14) equal to zero:

$$\lambda_S^C \equiv 1+\frac{\left(2\beta+(M-2)\gamma\right)\beta+(M-1)\left(\beta+(M-1)\gamma\right)\gamma}{M\Delta}>1 \tag{16}$$

The expression on the right-hand side is clearly greater than one and, reassuringly, reduces to the same solution as in Collie (2000) when products are homogeneous, $\gamma = \beta$. As the number of countries increases, the critical value of opportunity cost asymptotically approaches $\frac{2\beta}{2\beta-\gamma}$, which varies from two with homogeneous products to one when products are independent of each other, $\gamma = 0$. Differentiating the critical discount factor (16) with respect to the degree of substitutability between products yields:

$$\frac{\partial \lambda_S^C}{\partial \gamma} = \frac{\beta(M-1)}{M\Delta^2}\left[4\beta\left(\beta+(M-1)\gamma\right)+2M\gamma\left(2\beta+(M-2)\gamma\right)+\gamma^2\right]>0 \tag{17}$$

This is unambiguously positive and together with the other results in this section leads to the following proposition:

***Proposition 1.*** *Under Cournot oligopoly, the Nash equilibrium subsidy is positive if the opportunity cost of government revenue is less than the critical value,* $\lambda_S^C$*, and is decreasing in the opportunity cost of government revenue. Also, the critical value of opportunity cost is increasing in the degree of product substitutability,* $\partial\lambda_S^C/\partial\gamma > 0$.

An increase in the degree of substitutability between products strengthens the negative impact of the subsidy on the outputs and prices of the firms in the other countries. This increases the positive profit-shifting effect of the subsidy and reduces its negative terms of trade effect thereby strengthening the incentive for governments' to use subsidies. Hence, the critical value of opportunity cost required to make the Nash equilibrium subsidy equal to zero will be higher.

## 4. The Prohibition of Subsidies under Cournot Oligopoly

Having derived the Nash equilibrium in subsidies, the next step is to analyse the welfare effect of a uniform reduction in subsidies by all governments, and then to consider the welfare effect of the multilateral prohibition of subsidies. Since all the countries are assumed to be identical, they will all set the same subsidy and have the same level of welfare in the Nash equilibrium. Thus, as this symmetry will be maintained if there is a uniform reduction in subsidies by all countries, the welfare of each country can be considered to be a function of this common subsidy when looking at a uniform reduction in subsidies or the multilateral prohibition of subsidies.[9] Since the welfare of all countries is identical, the aggregate welfare effect on the customs union can be assessed by looking at the welfare of any one country.

The welfare effect on any country of a uniform reduction in subsidies can be assessed by differentiating (12) with respect to the common subsidy, which yields:

$$\frac{\partial W^C}{\partial s} = \left(p^C - c\right)\frac{\partial x^C}{\partial s} - \left(\lambda - 1\right)\left(x^C + s\frac{\partial x^C}{\partial s}\right) \tag{18}$$

The first term looks like the usual profit-shifting effect but, when considering a uniform reduction in subsidies, it is more accurate to describe it as the oligopolistic distortion effect. A uniform reduction in subsidies leads to a contraction in the output of the oligopolistic industry with a consequent increase in the deadweight loss from the oligopolistic distortion caused by price exceeding marginal cost. The second effect is the distortionary taxation effect. A uniform reduction in subsidies reduces the revenue required to finance the subsidy and thereby reduces the deadweight loss from the distortionary taxation. Note that there is no terms of trade effect with a uniform reduction in subsidies since trade is balanced and the common subsidy has an identical effect on all prices.[10] When lump-sum taxes are feasible, $\lambda = 1$, the distortionary taxation effect disappears leaving only the oligopolistic distortion effect. Then, provided price exceeds marginal cost, a uniform increase in subsidies will increase the welfare of all countries, and the joint welfare of all the countries will be maximised by a uniform subsidy that makes price equal to marginal cost. Hence, when lump-sum taxes are feasible, welfare at the Nash equilibrium in subsidies will be higher than welfare when subsidies are prohibited.

To evaluate this derivative at the Nash equilibrium in subsidies, the first-order condition for the Nash equilibrium (13) is used together with comparative static results derived from (11), which yields:

$$\frac{\partial W_N^C}{\partial s} = \frac{\left(M-1\right)x_N^C}{M\left(2\beta + \left(M-2\right)\gamma\right)}\left[\left(\beta - \gamma\right) - \left(\lambda - 1\right)M\gamma\right] \tag{19}$$

This derivative will be positive (negative) if the opportunity cost of government revenue is less (greater) than the critical value obtained by setting the term in square brackets equal to zero:

$$\lambda_N^C \equiv 1 + \frac{\beta - \gamma}{M\gamma} \geq 1 \tag{20}$$

When the opportunity cost of government revenue is greater (less) than the critical value, $\lambda > (<) \lambda_N^C$, the distortionary taxation effect will outweigh the oligopolistic distortion effect so a uniform reduction in subsidies will increase (decrease) the welfare of all countries. When $\lambda = \lambda_N^C$, the derivative (19) is equal to zero so the joint welfare of the countries is maximised and the outcome is Pareto efficient. This leads to the following proposition:

***Proposition 2.*** *Under Cournot Oligopoly, a uniform reduction in subsidies by all countries, evaluated at the Nash equilibrium in subsidies, will increase (decrease) the welfare of all countries if the opportunity cost of government revenue is greater (less) than* $\lambda > (<) \lambda_N^C$. *If* $\lambda = \lambda_N^C$ *then the Nash equilibrium in subsidies yields a Pareto efficient outcome!*

There are three special cases of this proposition that are worthy of special attention: Firstly, when products are homogeneous, $\gamma = \beta$, the critical value of opportunity cost is equal to one, $\lambda_N^C = 1$, so the Nash equilibrium is Pareto efficient if lump-sum taxes are feasible. At the Nash equilibrium, see (13), there is no terms of trade effect with homogeneous products and no distortionary taxation effect with lump-sum taxes so the profit-shifting effect must be equal to zero, which implies a Pareto efficient outcome with price equal to marginal cost. Secondly, when products are homogeneous and taxation is distortionary, opportunity cost is always higher than the critical value, $\lambda > \lambda_N^C = 1$, so a uniform reduction in subsidies will increase the welfare of all countries. At the Nash equilibrium, see (13), there is no terms of trade effect but there is a distortionary taxation effect and it is straightforward to show that $p_N^C - c > (\lambda - 1) s_N^C$, which implies that a country's subsidy will have a negative profit-shifting effect on the welfare of all other countries. This negative externality means that the Nash equilibrium subsidy is larger than the joint welfare maximising subsidy, and that a uniform reduction in subsidies will increase the welfare of all countries. Thirdly, when lump-sum taxes are feasible and products are differentiated, $\gamma < \beta$, a uniform reduction in subsidies will decrease the welfare of all countries since opportunity cost is less than the critical value, $\lambda = 1 < \lambda_N^C$. At the Nash equilibrium, there is no distortionary taxation effect and the terms of trade effect is negative with differentiated products so the profit-shifting effect will be positive. Hence, price will exceed marginal cost so a uniform reduction in subsidies will increase the oligopolistic distortion and reduce the welfare of all countries.

So far it has been shown that a uniform reduction in subsidies, evaluated at the Nash equilibrium, may increase the welfare of all countries, but this does not prove that the multilateral prohibition of subsidies may yield higher welfare for all countries than the Nash equilibrium in subsidies. To demonstrate that the multilateral prohibition of subsidies may be beneficial consider figure one, which shows the welfare of a country when subsidies are prohibited, $W_P^C$, and in the Nash equilibrium, $W_N^C$, as functions of the opportunity cost of government revenue. Obviously, welfare when subsidies are prohibited is independent of the opportunity cost of government revenue since with no subsidy to finance there is no deadweight loss from distortionary taxation. Above, it was shown that when lump-sum taxes are feasible, $\lambda = 1$, welfare in the Nash equilibrium will be higher than when subsidies are prohibited. When opportunity cost is equal to the critical value, $\lambda = \lambda_S^C$, the Nash equilibrium subsidy is equal to zero so welfare is obviously the same in the Nash equilibrium as when subsidies are prohibited. To determine the relative positions of the two curves for values of opportunity cost between one and $\lambda_S^C$, consider the slope of welfare in the Nash equilibrium. This is obtained by totally differentiating (12) with respect to opportunity cost and evaluating at the Nash equilibrium:

$$\frac{dW_N^C}{d\lambda} = \frac{\partial W_N^C}{\partial s}\frac{ds_N^C}{d\lambda} + \frac{\partial W_N^C}{\partial \lambda} = \frac{\partial W_N^C}{\partial s}\frac{ds_N^C}{d\lambda} - s_N^C x_N^C \tag{21}$$

At the critical value of opportunity cost, $\lambda = \lambda_S^C$, the Nash equilibrium subsidy is zero so the second term vanishes while the first term can be evaluated by substituting the critical value of opportunity cost from (16) into (19) and noting from (15) that $ds_N^C/d\lambda$ is negative, which yields:

$$\frac{dW_N^C}{d\lambda} = \frac{(M-1)x_N^C}{M\Delta}\left[2\beta^2 - 3\beta\gamma - (M-1)\gamma^2\right]\frac{ds_N^C}{\partial\lambda} \tag{22}$$

This derivative will be positive if the quadratic in square brackets is negative, which will be the case if $\gamma$ is greater than the larger root or less than smaller root of the quadratic. Since the smallest root is always negative, it can be ignored so the derivative will be positive (negative) if $\gamma$ is greater (less) than:

$$\hat{\gamma}^C \equiv \frac{-3+\sqrt{8M+1}}{2(M-1)}\beta \tag{23}$$

This critical value of product substitutability is shown in figure two as a function of the number of countries. When there are two countries $\hat{\gamma}^C \approx 0\cdot 56\beta$ and it is decreasing in the number of countries and asymptotically approaches zero as the number of countries goes to infinity.

For $\gamma > \hat{\gamma}^C$, welfare in the Nash equilibrium is positively sloped at the critical value of opportunity cost, $\lambda = \lambda_S^C$, which means that it must be 'U' shaped as shown in figure one. Hence, there must be some value of opportunity cost, $\lambda_P^C$, greater than one but less than the critical value of opportunity cost, $\lambda_S^C$, where welfare in the Nash equilibrium is equal to welfare when subsidies are prohibited, $W_N^C\left(\lambda_P^C\right) = W_P^C$. Thus, for values of opportunity cost in the range $\left(\lambda_P^C, \lambda_S^C\right)$ welfare of all countries is higher when subsidies are prohibited than in the Nash equilibrium. This leads to the following proposition:

***Proposition 3.*** *Under Cournot Oligopoly, for $\gamma > \hat{\gamma}^C$, there exists a range of values for the opportunity cost of government revenue, $\lambda_P^C > \lambda > \lambda_S^C$, where the Nash equilibrium subsidy is positive and aggregate welfare would be higher if subsidies were prohibited than in the Nash equilibrium in subsidies.*

Provided the products are sufficiently close substitutes, there exists a range of values for opportunity cost where prohibition of subsidies is beneficial for all countries. The multilateral prohibition of subsidies prevents a subsidy war that leads to a prisoner's dilemma type of outcome. The result in Collie (2000) for homogeneous products is a special case of this proposition, although Collie considers non-linear demand functions.

For $\gamma < \hat{\gamma}^C$, since $\partial W_N^C / \partial s$ is positive at the critical value of opportunity cost, $\lambda = \lambda_S^C$, it follows from proposition two that $\lambda_S^C < \lambda_N^C$ so $\partial W_N^C / \partial s$ is positive for all values of opportunity cost between one and the critical value $\lambda_S^C$. Hence, both the first and second terms in (21) will be negative and so welfare in the Nash equilibrium will be negatively sloped for all values of opportunity cost between one and the critical value $\lambda_S^C$. Therefore, as shown in figure one, welfare in the Nash equilibrium will be higher than welfare when subsidies are prohibited for all values of opportunity cost where the Nash equilibrium subsidy is positive, $1 < \lambda < \lambda_S^C$. This leads to the following proposition:

***Proposition 4.*** *Under Cournot oligopoly, for $\gamma < \hat{\gamma}^C$, whenever the Nash equilibrium subsidy is positive, $1 \leq \lambda < \lambda_S^C$, aggregate welfare would be lower if subsidies were prohibited than in the Nash equilibrium in subsidies.*

When products are sufficiently differentiated, the multilateral prohibition of subsidies will never be beneficial as the subsidy war does not lead to a prisoner's dilemma type of outcome. Some intuition for the importance of product differentiation in these last two propositions can be obtained by looking at the externality that one country's subsidy imposes on the other countries. A country's subsidy has two main effects on the welfare of the other countries: a positive terms of trade effect and a negative profit-shifting effect. When the products are sufficiently close substitutes, the negative profit-shifting effect outweighs the positive terms of trade effect so the subsidy causes a negative externality and the welfare of all countries will be increased by a multilateral prohibition of subsidies. When the products are sufficiently differentiated, the positive terms of trade effect outweighs the negative terms of trade effect so the subsidy causes a positive externality and the welfare of all countries will be decreased by a multilateral prohibition of subsidies. In fact, in this case, a uniform increase in subsidies would increase the welfare of all countries.

The practical relevance of these propositions can now be assessed by calculating the critical values of opportunity cost for various parameter values then comparing these critical values with empirical estimates of the opportunity cost of government revenue. Based upon the review of empirical estimates in Snow and Warren (1996), Collie (2000) argues that $1 \cdot 2$ is a perfectly plausible value for the opportunity cost of government revenue, and this number should be borne in mind. The critical value of opportunity cost $\lambda_S^C$ comes from (16) and $\lambda_N^C$

comes from (20), while $\lambda_P^C$ is obtained by choosing the appropriate solution of the equation $W_N^C\left(\lambda_P^C\right)=W_P^C$, which can be solved explicitly using *Mathematica*, but this solution is not presented as it is extremely messy. These critical values of opportunity cost only depend upon the degree of product substitutability, $\gamma/\beta$, and the number of countries, $M$, so can best be illustrated by plotting them as functions of the number of countries for a given value of product substitutability.

Figures three, four, and five show plots of the critical values of opportunity cost for high, medium, and low degrees of product substitutability, $\gamma/\beta$, equal to $9/10$, $3/4$, and $1/2$ respectively. Recall that the Nash equilibrium subsidy is positive for $\lambda < \lambda_S^C$ and a uniform reduction in subsidies will increase welfare for $\lambda > \lambda_N^C$ while the prohibition of subsidies will increase the welfare of all countries for $\lambda_P^C < \lambda < \lambda_S^C$. With both high and medium degrees of product substitutability, see figures three and four, there is a wide range of values for opportunity cost where the Nash equilibrium subsidy is positive and the multilateral prohibition of subsidies will increase the welfare of all countries. In both cases, the range of values includes the plausible value of opportunity cost, $\lambda = 1\cdot 2$, when there are more than three or four countries. With a low degree of product substitutability, see figure five, the range of values is much smaller but includes the plausible value of opportunity cost when there are more than half a dozen countries. When there are two or three countries, there is no range of values where the Nash equilibrium is positive and the multilateral prohibition of subsidies increases welfare as the degree of product substitutability is less than the critical value, $\hat{\gamma}^C \leq 1/2$ for $M \leq 3$. In conclusion, under Cournot oligopoly, the Nash equilibrium subsidy is positive and the multilateral prohibition of subsidies would increase the welfare of all countries for a reasonably large range of plausible parameter values.

## 5. The Model with Bertrand Oligopoly

Having analysed the model under Cournot oligopoly in great detail, the case of Bertrand oligopoly will now be analysed briefly. At a Bertrand equilibrium, the $M$ firms independently and simultaneously set prices to maximise their profits given the subsidies set by the national governments at the first stage of the game. With linear demand functions, the Bertrand equilibrium will be unique and the second order conditions for profit maximisation are bound to be satisfied since profit functions are concave in prices. Linear demand functions also imply that prices will be strategic complements so the reaction functions will be upward sloping. Assuming an interior solution, where all firms produce positive quantities, maximising profits (6) with respect to price yields the first-order conditions for a Bertrand equilibrium are:

$$\frac{\partial \pi_i}{\partial p_i} = x_i + \left( p_i - c + s_i \right) \frac{\partial x_i}{\partial p_i} = 0 \qquad (24)$$

Using the demand function (4) and noting that $\partial x_i / \partial p_i = -bM/\Phi$ yields a system of $M$ simultaneous equations:

$$2bp_i - \gamma \sum_{j \neq i} p_j = \alpha \left( \beta - \gamma \right) + b\left( \alpha - c \right) \qquad i = 1, \ldots, M \qquad (25)$$

Then, solving these simultaneous equations yields the Bertrand equilibrium prices as functions of the subsidies set by the national governments:

$$p_i^B = \frac{\left( \beta - \gamma \right)\alpha + bc}{2\beta + \left( M - 3 \right)\gamma} - \frac{b}{\Lambda}\left[ \left( 2\beta + \left( M - 2 \right)\gamma \right) s_i + \gamma \sum_{j \neq i} s_j \right] \qquad (26)$$

where $\Lambda \equiv \left( 2\beta + \left( M - 3 \right)\gamma \right)\left( 2\beta + \left( 2M - 3 \right)\gamma \right) > 0$ and the superscript $B$ is used to denote the Bertrand equilibrium. All subsidies have the effect of reducing the Bertrand equilibrium price set by the firms but the domestic subsidy has a larger effect on price than foreign subsidies. Substituting the Bertrand equilibrium prices into the ordinary demand function (4) yields the Bertrand equilibrium outputs as functions of the subsidies:

$$x_i^B = \frac{bM(\beta-\gamma)(\alpha-c)}{\Phi\left(2\beta+(M-3)\gamma\right)} + \frac{bM}{\Phi\Lambda}\left[\left(b^2+(\beta-\gamma)\left(\beta+(M-1)\gamma\right)\right)s_i - \gamma b\sum_{j\neq i} s_j\right] \tag{27}$$

The output of a firm is increasing in the domestic subsidy and decreasing in foreign subsidies. Since the model is symmetric, if all governments give the same subsidy, $s_i = s$, then all firms set the same price and sell the same quantity of output. These symmetric Bertrand equilibrium price and output are:

$$p^B = \frac{\alpha(\beta-\gamma)+b(c-s)}{2\beta+(M-3)\gamma} \qquad x^B = \frac{Mb(\alpha-c+s)}{\left(\beta+(M-1)\gamma\right)\left(2\beta+(M-3)\gamma\right)} \tag{28}$$

An increase in this common subsidy will reduce the Bertrand equilibrium price and increase the output sold by the firms.

## 6. Nash Equilibrium Subsidies under Bertrand Oligopoly

At the first stage of the game, the national governments each independently and simultaneously set their subsidies to maximise national welfare (12) realising that the firms will compete as Bertrand oligopolists in the second stage of the game with outputs and prices given by (26) and (27). The first-order conditions for the Nash equilibrium in subsidies under Bertrand oligopoly is exactly the same as under Cournot oligopoly, (13). Imposing symmetry and using (26) and (27) to evaluate the derivatives in (13) yields an implicit solution for the Nash equilibrium subsidy under Bertrand oligopoly:

$$s_N^B = \frac{\Phi}{\lambda M^2}\frac{b^2+(\beta-\gamma)(b+M\gamma)-(\lambda-1)M\Lambda}{b^2+(\beta-\gamma)\left(\beta+(M-1)\gamma\right)}x_N^B \tag{29}$$

With differentiated products, $\gamma < \beta$, the Nash equilibrium subsidy under Bertrand oligopoly is unambiguously positive when lump-sum taxes are feasible, $\lambda = 1$.[11] Given the well known result in Eaton and Grossman (1986), the fact that the Nash equilibrium subsidy is positive under both Cournot and Bertrand Oligopoly may be surprising to some readers, but a production subsidy is very different to an export subsidy since domestic consumers benefit from the lower price with a production subsidy. By differentiating (29) it can be shown that the Nash equilibrium subsidy is decreasing in the opportunity cost of government revenue:

$$\frac{ds_N^B}{d\lambda} = \frac{-\Phi^2\left(2\beta + (M-3)\gamma\right)}{\lambda^2 M b^2 (\alpha - c)(\beta - \gamma)} \cdot \frac{b^2 + (\beta - \gamma)(b + M\gamma) + M\Lambda}{b^2 + (\beta - \gamma)\left(\beta + (M-1)\gamma\right)} \left(x_N^B\right)^2 < 0 \qquad (30)$$

Hence, with distortionary taxation, the Nash equilibrium subsidy will only be positive if the opportunity cost of government revenue is less than the critical value $\lambda_S^B$, which is obtained by setting the numerator of (29) equal to zero:

$$\lambda_S^B = 1 + \frac{b^2 + (\beta - \gamma)(b + M\gamma)}{M\Lambda} > 1 \qquad (31)$$

Although this critical value of opportunity cost is greater than one, it asymptotically approaches one as the number of countries becomes very large so the Nash equilibrium subsidy will be negative when the number of countries is sufficiently large if lump-sum taxes are not feasible, $\lambda > 1$. To compare the critical value of opportunity cost under Bertrand oligopoly with that under Cournot oligopoly, subtract (31) from (16), which after much simplification yields:

$$\lambda_S^C - \lambda_S^B = \frac{(M-1)\gamma^2}{M\Delta\Lambda}\left[4(2M-1)(\beta - \gamma)\left(\beta + (M-1)\gamma\right) + \left(2M^2(M-1)+1\right)\gamma^2\right] > 0 \quad (32)$$

Since this is unambiguously positive, the critical value of opportunity cost is lower under Bertrand oligopoly than under Cournot oligopoly, $\lambda_S^B < \lambda_S^C$. The results of this section are summarised in the following proposition:

***Proposition 5.*** *The Nash equilibrium subsidy under Bertrand oligopoly is positive if the opportunity cost of government revenue is less than the critical value,* $\lambda_S^B$*, and is decreasing in the opportunity cost of government revenue. The critical value of opportunity cost is lower under Bertrand oligopoly than under Cournot oligopoly.*

Comparing this proposition with proposition one, it can be seen that the results under Bertrand oligopoly are qualitatively similar to those under Cournot oligopoly.[12] Since the price-cost margin under Bertrand oligopoly is lower than under Cournot oligopoly, the profit-shifting effect of the subsidy will be lower hence the incentive to use a subsidy will be lower and therefore the critical value of opportunity cost will be lower under Bertrand oligopoly.[13]

## 7. Prohibition of Subsidies under Bertrand Oligopoly

The next step is to analyse the welfare effect of a uniform reduction in subsidies by all governments, and this can be done by differentiating (12) with respect to the common subsidy:

$$\frac{\partial W^B}{\partial s} = \left(p^B - c\right)\frac{\partial x^B}{\partial s} - \left(\lambda - 1\right)\left(x^B + s\frac{\partial x^B}{\partial s}\right) \tag{33}$$

To evaluate this derivative at the Nash equilibrium in subsidies, the first-order condition for the Nash equilibrium (13) is used together with comparative static results derived from (28), which yields:

$$\frac{\partial W_N^B}{\partial s} = \frac{(M-1)bx_N^B}{M} \cdot \frac{(\beta-\gamma)-(\lambda-1)M\gamma}{b^2+(\beta-\gamma)(\beta+(M-1)\gamma)} \tag{34}$$

This derivative will be positive (negative) if the opportunity cost of government revenue is less (greater) than the critical value of opportunity cost obtained by setting the numerator equal to zero:

$$\lambda_N^B \equiv 1 + \frac{\beta-\gamma}{M\gamma} \tag{35}$$

Surprisingly, this is exactly the same critical value of opportunity cost as under Cournot oligopoly, $\lambda_N^C = \lambda_N^B$. This result leads to the following proposition:

***Proposition 6.*** *Under Bertrand Oligopoly, a uniform reduction in subsidies by all countries, evaluated at the Nash equilibrium in subsidies, will increase (decrease) the welfare of all countries if the opportunity cost of government revenue is greater (less) than* $\lambda > (<) \lambda_N^B$. *If* $\lambda = \lambda_N^B$ *then the Nash equilibrium in subsidies yields a Pareto efficient outcome!*

Since the critical value of opportunity cost is identical under both Cournot and Bertrand oligopoly, this proposition is exactly the same as proposition two. Although the critical value of opportunity cost is identical under both Cournot and Bertrand oligopoly, the magnitudes of the two derivatives (19) and (34) are not identical.

This proposition shows that a uniform reduction in subsidies, evaluated at the Nash equilibrium, may increase the welfare of all countries but does not prove that the multilateral prohibition of subsidies may increase the welfare of all countries. To prove that the multilateral prohibition of subsidies may be beneficial consider again figure one, which shows the welfare of a country when subsidies are prohibited, $W_P^B$, and in the Nash equilibrium in subsidies, $W_N^B$, as functions of the opportunity cost of government revenue.[14] Welfare when subsidies are prohibited is independent of opportunity cost and, when lump-sum taxes are feasible, welfare in the Nash equilibrium is higher than welfare when subsidies are prohibited. When $\lambda = \lambda_S^B$, the Nash equilibrium subsidy is equal to zero so welfare in the Nash equilibrium is equal to welfare when subsidies are prohibited. To determine the relative position of the two curves between one and $\lambda_S^B$ consider the slope of welfare in the Nash equilibrium. This is obtained by differentiating (12) with respect to opportunity cost and evaluating at the Nash equilibrium:

$$\frac{dW_N^B}{d\lambda} = \frac{\partial W_N^B}{\partial s}\frac{ds_N^B}{d\lambda} + \frac{\partial W_N^B}{\partial \lambda} = \frac{\partial W_N^B}{\partial s}\frac{ds_N^B}{d\lambda} - s_N^B x_N^B \tag{36}$$

At the critical value of opportunity cost, $\lambda = \lambda_S^B$, the Nash equilibrium subsidy is equal to zero so the second term vanishes while the first term can be evaluated by substituting the critical value of opportunity cost from (31) into (34) and noting from (30) that $ds_N^B/d\lambda$ is negative, which yields:

$$\frac{dW_N^B}{d\lambda} = \frac{(M-1)bx_N^B}{M\Lambda}\left[2\beta - 3\gamma\right]\frac{ds_N^B}{\partial\lambda} \tag{37}$$

This derivative will be positive (negative) if the term in square brackets is negative (positive) which will be the case if $\gamma$ is greater (less) than the critical value $\hat{\gamma}^B \equiv 2\beta/3$. The critical value of product substitutability is higher under Bertrand oligopoly than under Cournot oligopoly. Also, the critical value of product substitutability is independent of the number of countries under Bertrand oligopoly whereas it is decreasing in the number of countries under Cournot oligopoly.

For $\gamma > \hat{\gamma}^B$, welfare in the Nash equilibrium is positively sloped at the critical value of opportunity cost, $\lambda = \lambda_S^B$, which means that it must be 'U' shaped as shown in figure one. Hence, there must be some value of opportunity cost, $\lambda_P^B$, greater than one but less than the critical value, $\lambda_S^B$, where welfare in the Nash equilibrium is equal to welfare when subsidies are prohibited, $W_N^B\left(\lambda_P^B\right) = W_P^B$. Thus, for values of opportunity cost in the range $\left(\lambda_P^B, \lambda_S^B\right)$, the welfare of all countries is higher when subsidies are prohibited than in the Nash equilibrium. This leads to the following proposition:

***Proposition** 7. Under Bertrand Oligopoly, for $\gamma > \hat{\gamma}^B = 2\beta/3$, there exists a range of values for the opportunity cost of government revenue, $\lambda_P^B > \lambda > \lambda_S^B$, where the Nash equilibrium subsidy is positive and aggregate welfare would be higher if subsidies were prohibited than in the Nash equilibrium in subsidies.*

This proposition is qualitatively similar to proposition three. Provided the products are sufficiently close substitutes, there exists a range of values for opportunity cost where the multilateral prohibition of subsidies is beneficial as it prevents a subsidy war that leads to a prisoner's dilemma type of outcome.

For $\gamma < \hat{\gamma}^B$, since $\partial W_N^B / \partial s$ is positive at the critical value of opportunity cost, $\lambda = \lambda_S^B$, it follows from proposition two that $\lambda_S^B < \lambda_N^B$ so $\partial W_N^B / \partial s$ is positive for all values of opportunity cost between one and the critical value $\lambda_S^B$. Hence, both the first and second terms in (36) will be negative and so welfare in the Nash equilibrium will be negatively sloped for all values of opportunity cost between one and the critical value $\lambda_S^B$. Therefore, as shown in figure one, welfare in the Nash equilibrium will be higher than welfare when subsidies are prohibited for all values of opportunity cost where the Nash equilibrium subsidy is positive, $1 < \lambda < \lambda_S^B$. This leads to the following proposition:

***Proposition 8.*** *Under Bertrand oligopoly, for* $\gamma < \hat{\gamma}^B = 2\beta/3$, *whenever the Nash equilibrium subsidy is positive,* $1 \leq \lambda < \lambda_S^B$, *aggregate welfare would be lower if subsidies were prohibited than in the Nash equilibrium in subsidies.*

This proposition is qualitatively similar to proposition four. When products are sufficiently differentiated, the multilateral prohibition of subsidies will never be beneficial as the subsidy war does not lead to a prisoner's dilemma type of outcome.

The practical relevance of these propositions and the quantitative differences between the case of Cournot and Bertrand oligopoly can now be assessed by calculating the critical values of opportunity cost for various parameter values. The critical value of opportunity cost $\lambda_S^B$ comes from (31) and $\lambda_N^B$ comes from (35) while $\lambda_P^B$ is obtained by choosing the appropriate solution of the equation $W_N^B\left(\lambda_P^B\right) = W_P^B$ using *Mathematica*. Figures six and seven show plots of the critical values of opportunity cost for high and medium degrees of product substitutability, $\gamma/\beta$, equal to $9/10$ and $3/4$ respectively.[15] Recall that the Nash equilibrium subsidy is positive for $\lambda < \lambda_S^B$ and a uniform reduction in subsidies will increase welfare for $\lambda > \lambda_N^B$ while the prohibition of subsidies will increase the welfare of all countries for $\lambda_P^B < \lambda < \lambda_S^B$.

With a high degree of product substitutability, see figure six, there is a range of values for opportunity cost where the Nash equilibrium subsidy is positive and the multilateral prohibition of subsidies will increase the welfare of all countries, but this range of values is below the plausible value of opportunity cost, $\lambda = 1 \cdot 2$. With a medium degree of product substitutability, see figure seven, there is a very narrow range of values where the Nash equilibrium is positive and the multilateral prohibition of subsidies will increase the welfare of all countries, but this range of values is below the plausible value of opportunity cost when there are more than two countries.[16] Figure eight shows the difference $\lambda_P^C - \lambda_P^B$ as a function of the number of countries and the degree of product substitutability, and it proves that the critical value of opportunity cost is larger under Cournot oligopoly than under Bertrand oligopoly, $\lambda_P^C > \lambda_P^B$. In conclusion, under Bertrand oligopoly, the Nash equilibrium subsidy is unlikely to be positive for any reasonable parameter values.

## 8. Conclusions

The multilateral prohibition of state aid in an integrated market has been analysed in a model where firms produce nationally differentiated products and compete as Cournot or Bertrand oligopolists. There are many results in this paper but the two main results are: Firstly, under both Cournot and Bertrand oligopoly, if the products are sufficiently close substitutes then there exists a range of values for the opportunity cost of government revenue where the Nash equilibrium subsidy is positive and where the multilateral prohibition of subsidies will increase the welfare of all countries. In this case, the multilateral prohibition of subsidies prevents a subsidy 'war' that leads to a prisoner's dilemma type of outcome. Secondly, under both Cournot and Bertrand oligopoly, if products are sufficiently differentiated and the Nash equilibrium subsidy is positive then the multilateral prohibition of subsidies will reduce the welfare of all countries. In this case, a subsidy 'war' does not lead to a prisoner's dilemma type of outcome but to an outcome that benefits all the countries so the multilateral prohibition of subsidies would be harmful.

In Collie (2000), with homogeneous products, there always exists a range of values for the opportunity cost of government revenue where the Nash equilibrium subsidy is positive and the multilateral prohibition of subsidies would increase the welfare of all countries. Here, it has been shown that this result is not robust to the introduction of product differentiation into the model. The intuition for this result is that when products are sufficiently differentiated, the negative effect of a country's subsidy on the profits of foreign firms is outweighed by the positive effect on foreign consumer surplus as a result of cheaper imports; hence, the subsidy causes a positive rather than a negative externality on foreign countries. The Nash equilibrium subsidy is lower than the joint welfare maximising subsidy, and a multilateral reduction of subsidies would actually reduce the welfare of all countries. This result suggests that European Commission should take into account the degree of product substitutability when assessing state aid cases, and it implies that state aid will be most harmful in industries where the products of different firms are close substitutes such as in the steel or the coal industries.

Assuming Bertrand oligopoly rather than Cournot oligopoly, does not alter the results in any qualitative manner and the Nash equilibrium subsidy will be positive provided the opportunity cost of government revenue is sufficiently low, which may be surprising given the well-known result of Eaton and Grossman (1986) about export subsidies. However, this should not be surprising as a production subsidy, unlike an export subsidy, reduces the prices paid by domestic consumers thereby increasing consumer surplus. Although there is no qualitative difference in the results under Cournot and Bertrand oligopoly, there is a quantitative difference. The critical value of opportunity cost below which the Nash equilibrium subsidy is positive is larger under Cournot oligopoly than under Bertrand oligopoly, and the Nash equilibrium subsidy under Bertrand oligopoly is unlikely to be positive for any plausible value of opportunity cost.

Of most significance for policy, this paper demonstrates the fundamental importance of the opportunity cost of government revenue for the welfare analysis of state aid, and this may be relevant to the European Commission's proposed block exemption of state aid. If lump-sum taxes were feasible then production subsidies would have a benevolent effect by reducing the oligopolistic distortion. However, with plausible values of opportunity cost, this paper has shown that production subsidies will either not be used under Bertrand oligopoly or should be prohibited under Cournot oligopoly. Thus, the European Commissions proposal to have a block exemption for seemingly benevolent forms of state aid (subsidies for SMEs, R&D, environmental protection, employment and training, and regional aid) may be misguided as it ignores the negative welfare effect caused by seemingly benevolent forms of state aid when account is taken of the opportunity cost of government revenue.

For plausible values of the opportunity cost of government revenue, this paper has shown that there is a strong case for prohibiting a benevolent form of state aid even when governments are welfare maximisers. Obviously, if governments do not maximise welfare and/or they give less benevolent forms of state aid (for example, subsidies to fixed costs that keep inefficient firms in business) then the case for the prohibition of state aid will be strengthened.

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**State Aid to the Manufacturing Sector as a Percentage of Value-Added**

| % | 1993 | 1994 | 1995 | 1996 | 1997 |
|---|---|---|---|---|---|
| EUR 12 | 3.8 | 3.5 | 3.2 | 2.9 | 2.6 |
| EUR 15 | N/A | N/A | 3.1 | 2.8 | 2.5 |

| % | 1993-95 | 1995-97 |
|---|---|---|
| Austria | N/A | 1.5 |
| Belgium | 2.5 | 2.4 |
| Denmark | 2.7 | 3.0 |
| Germany | 4.4 | 3.1 |
| Greece | 5.2 | 5.6 |
| Spain | 2.1 | 3.0 |
| Finland | N/A | 1.6 |
| France | 2.1 | 2.0 |
| Ireland | 2.4 | 2.2 |
| Italy | 6.1 | 5.3 |
| Luxembourg | 2.2 | 2.3 |
| Netherlands | 1.1 | 1.2 |
| Portugal | 2.7 | 2.8 |
| Sweden | N/A | 1.0 |
| United Kingdom | 0.8 | 0.9 |
| EUR 12 | 3.5 | 2.9 |
| EUR 15 | N/A | 2.8 |

Source: Seventh Survey on State Aid in the European Union in the Manufacturing and Certain Other Sectors, SEC(99)148.

## Endnotes

[1] The Treaty of Amsterdam, which comes into force on the 1st May 1999, renumbers the Articles of the EC Treaty including those relevant to state aid, Articles 87, 88, and 89, which were formerly Articles 92, 93, and 94 respectively.

[2] Details of the state aid regulations are given in Commission the European Community (1995), and an explanation of these rules is provided in Commission of the European Communities (1997). For recent developments in state aid policy see Commission of the European Communities (1998 and 1999a). The most recent survey of state aid in the EC is the seventh survey, Commission of the European Communities (1999b). Behboodi (1994) discusses the European Community's supranational approach to the regulation of subsidies in contrast to the multilateral approach under the GATT/WTO.

[3] All consumption of the exported product occurs in a third country, which benefits from the export subsidy war between the two exporting countries. In fact, world welfare will be higher with the export subsidy war than under free trade as the export subsidies reduce the oligopolistic distortion. A possible rationale for the GATT/WTO prohibition of export subsidies is suggested by Collie (1997) in multi-country version of the Brander-Spencer model with distortionary taxation.

[4] The relevance of the opportunity cost of government revenue for the assessment of interventions to correct market failures is mentioned by Gilchrist and Deacon (1990) in their discussion of state aid policy.

[5] Assuming an integrated market rather than $M$ segmented markets seems to be the obvious way to model the Single Market where most barriers to arbitrage have been eliminated. However, with symmetry and no barriers to trade, assuming instead that the Single Market consisted of $M$ segmented markets would not affect the results in any way. Each firm would have the same market share in all segmented markets as it would have in the integrated market so the price will be the same in all segmented markets as in the integrated market.

[6] Although the analysis will be carried out in a partial equilibrium framework, it could easily carried out in an explicitly general equilibrium framework by adding a competitive numeraire good produced under constant returns to scale.

[7] This utility function is borrowed from an example in Vives (1985) who uses it to compare the efficiency of Cournot and Bertrand equilibria.

[8] Judging from the results in Collie (2000), the results of this paper would not be altered in any qualitative sense by allowing non-linear demand functions but the analysis of non-linear demand with differentiated products would make the analysis very messy.

[9] The multilateral prohibition of subsidies can be analysed as a uniform reduction in subsidies from the Nash equilibrium subsidy down to zero in all countries.

[10] There is a negative terms of trade effect when a country increases its own subsidy, as in the Nash equilibrium, but this is offset by a positive terms of trade effect when the other countries increase their subsidies making imports cheaper.

[11] With homogeneous products, the Nash equilibrium subsidy is equal to zero since there is no incentive to use a subsidy for profit-shifting as firms set price equal to marginal cost. This is consistent with Eaton and Grossman (1986) who show in their proposition five that the optimal production tax/subsidy for a country is equal to zero under Bertrand oligopoly when products are homogeneous and marginal costs are constant.

[12] Except that the critical value of opportunity cost under Bertrand oligopoly is not always increasing in the degree of product substitutability as it is under Cournot oligopoly.

[13] In a fairly general model, Vives (1985) proves that prices will generally be lower under Bertrand oligopoly than under Cournot oligopoly so this result is probably fairly robust.

[14] The labelling in figure one is for the case of Cournot oligopoly considered above so now when analysing the case of Bertrand oligopoly the *C* superscripts should be replaced with *B* superscripts.

[15] With a low degree of product substitutability, equal to one-half, there is no range of values where the Nash equilibrium is positive and the multilateral prohibition of subsidies will increase the welfare of all countries as the degree of product substitutability is less than the critical value of two-thirds. Hence, this case is not plotted for the case of Bertrand oligopoly.

[16] The narrow range of values for opportunity cost is due to the fact that the degree of product substitutability is close to the critical value of two-thirds.

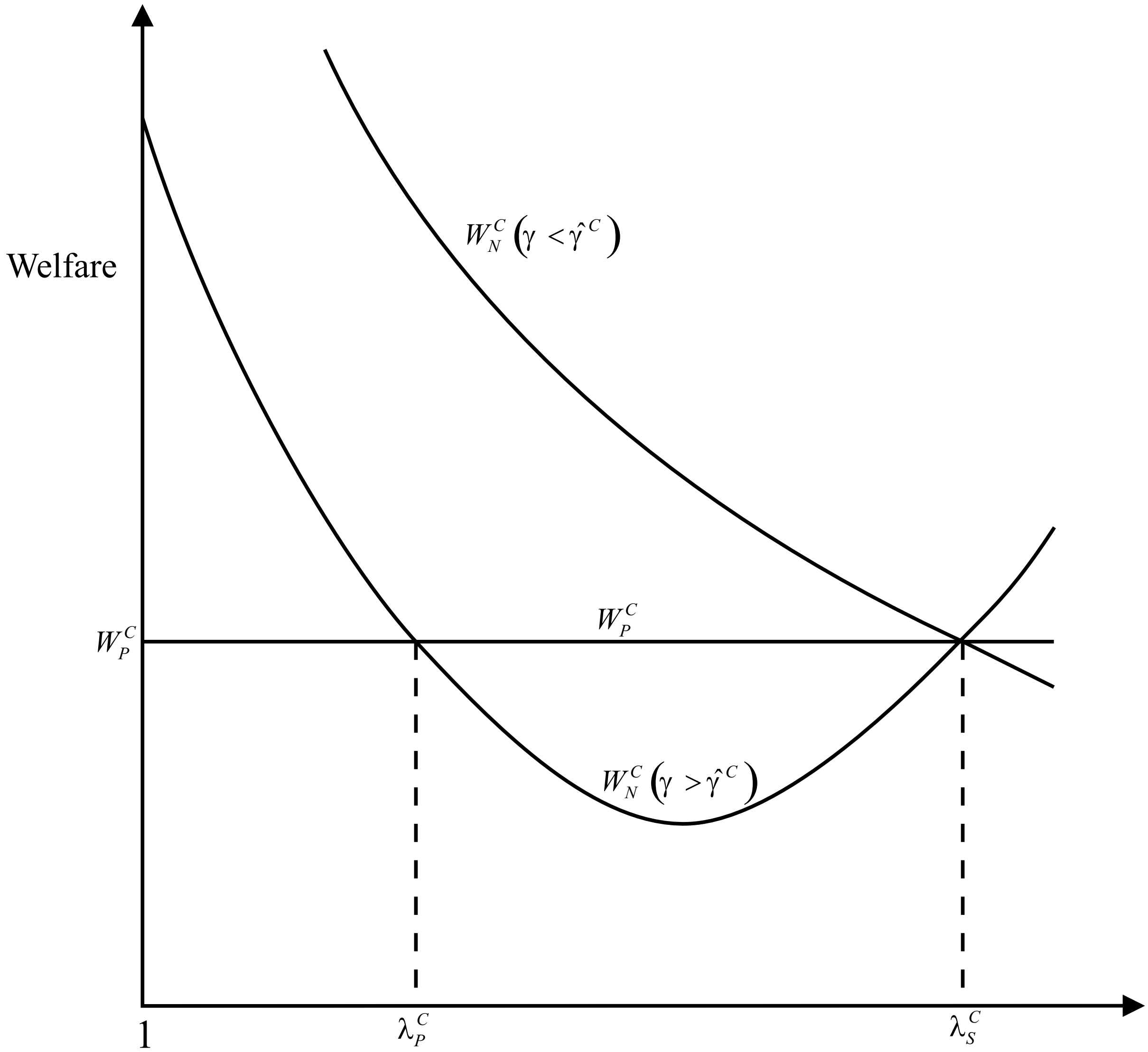


Figure 1: Welfare Comparison of Nash Equilibrium with Prohibition of Subsidies

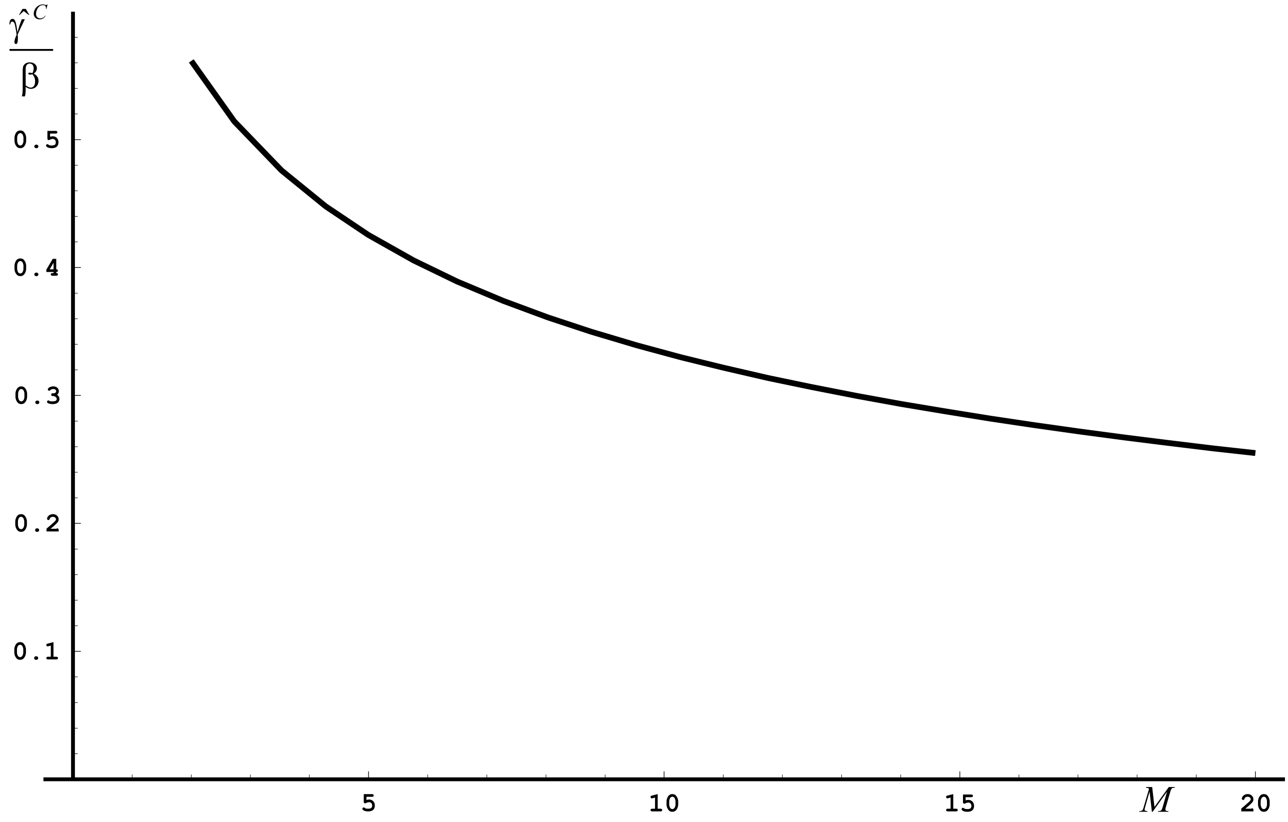

Figure 2: Critical Value of Product Substitutability under Cournot Oligopoly

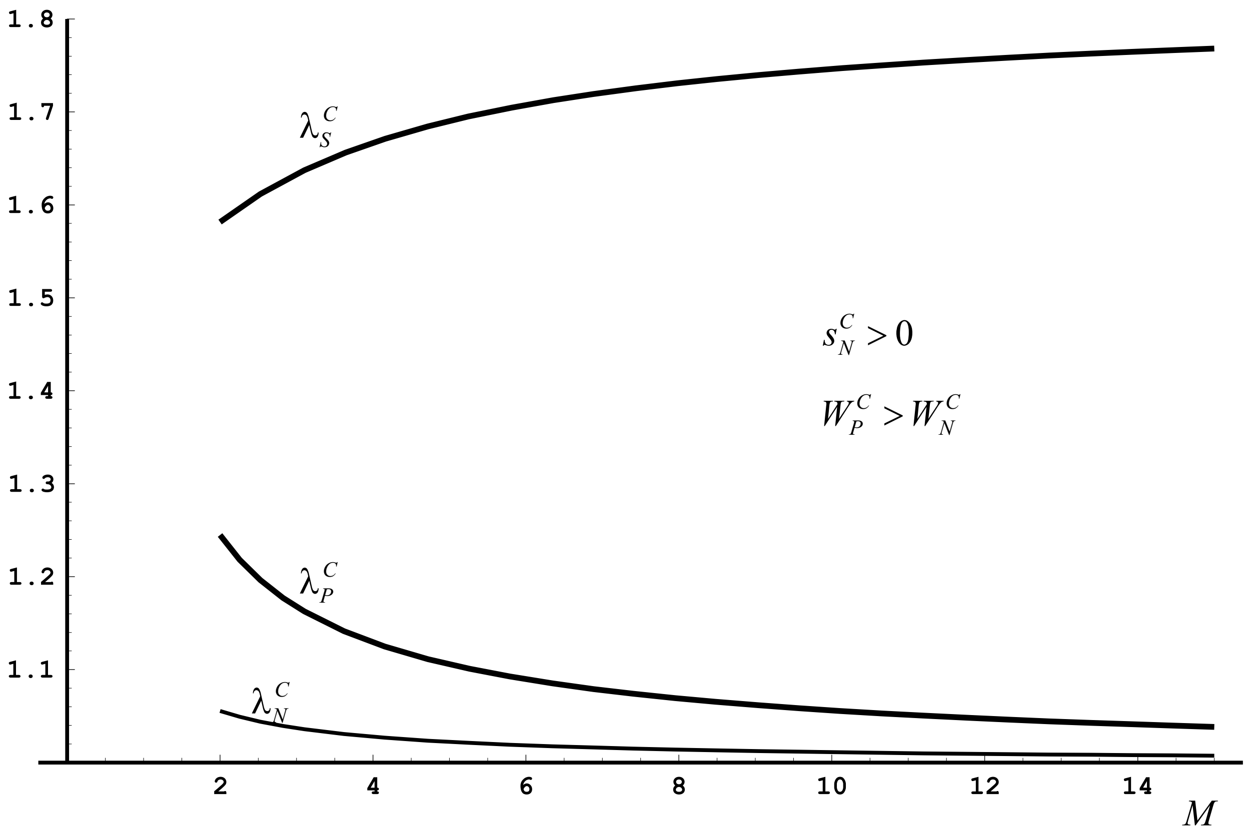

Figure 3: Critical Values of Opportunity Cost for γ/β=9/10 under Cournot Oligopoly

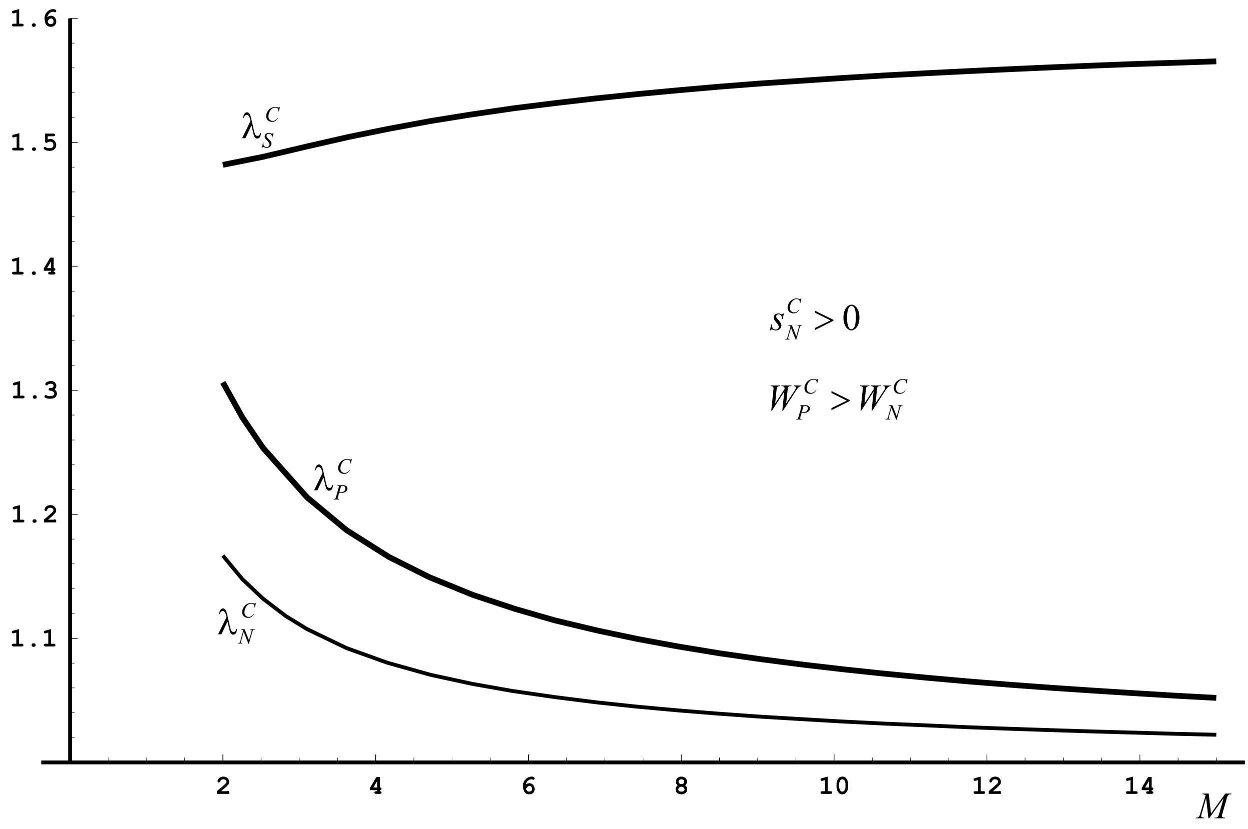


Figure 4: Critical Values of Opportunity Cost for γ/β=3/4 under Cournot Oligopoly

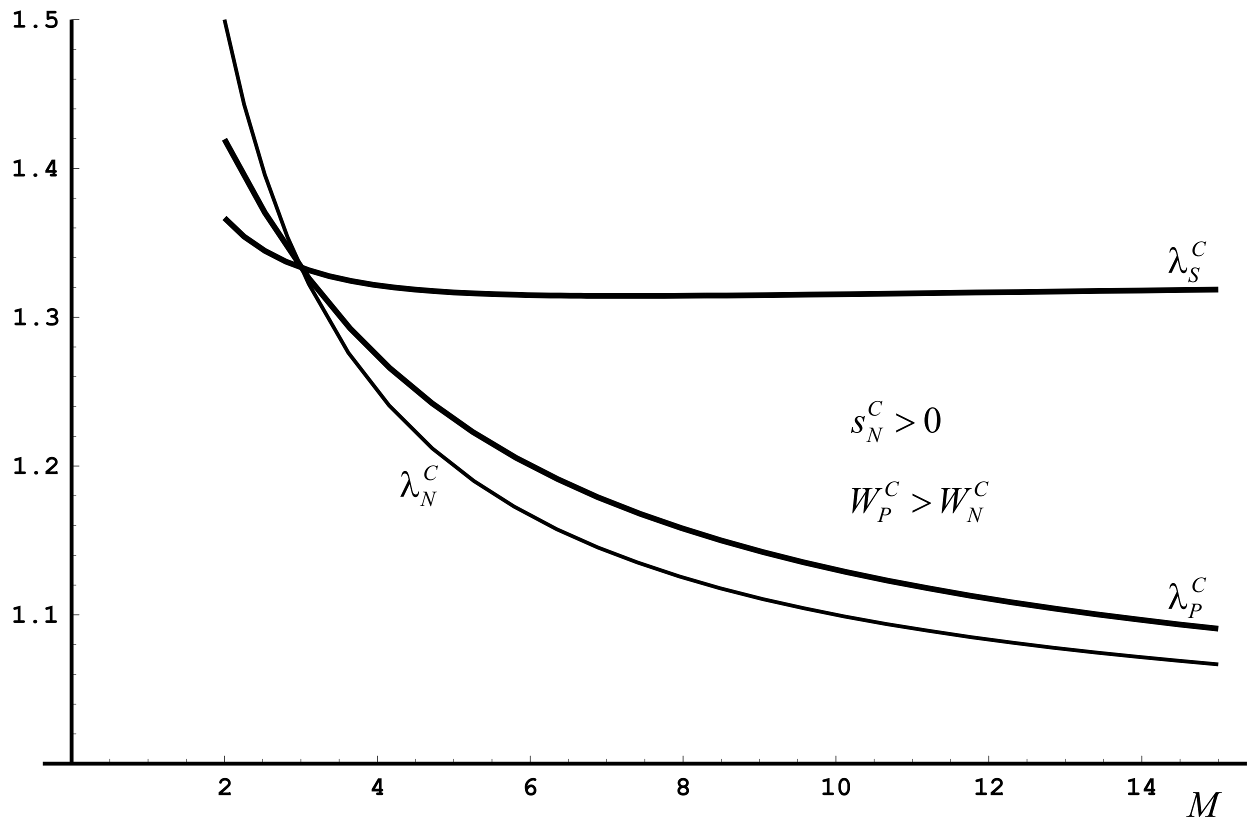


Figure 5: Critical Values of Opportunity Cost for γ/β=1/2 under Cournot Oligopoly

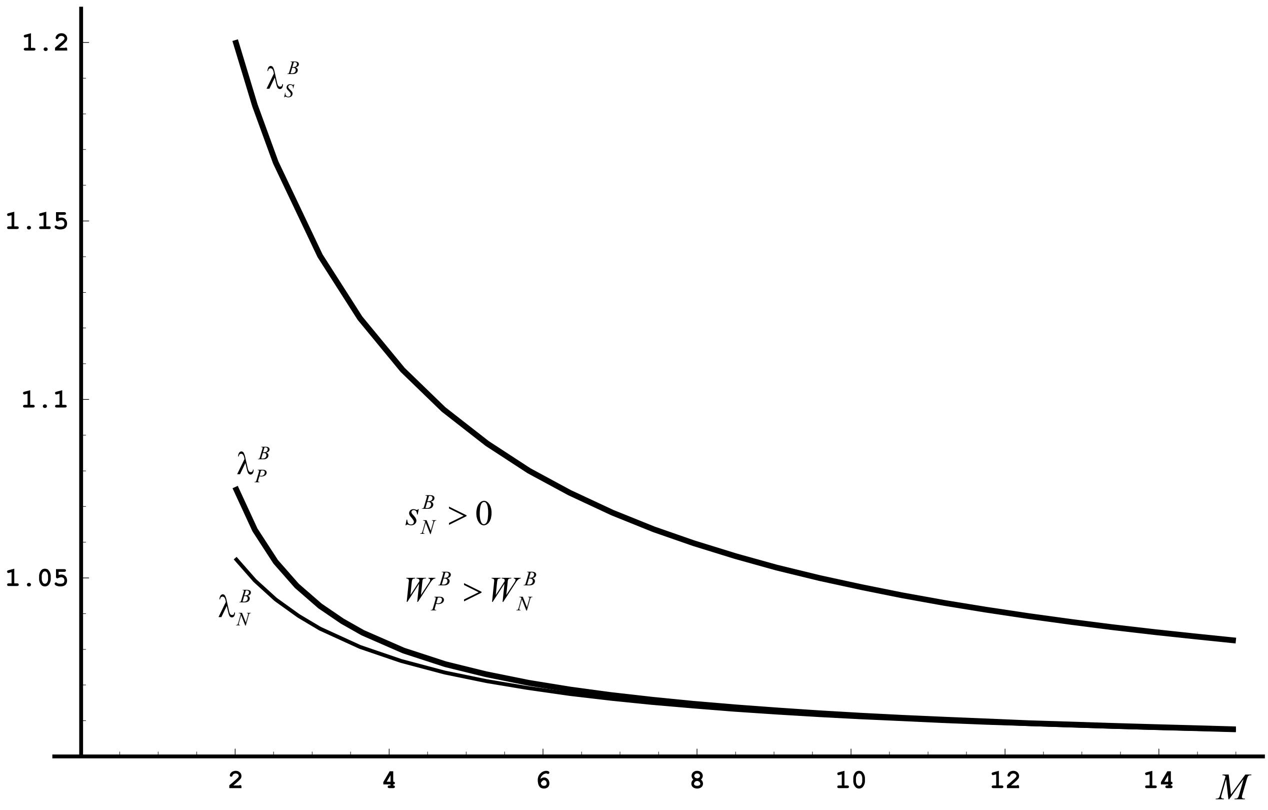


Figure 6: Critical Values of Opportunity Cost for γ/β=9/10 under Bertrand Oligopoly

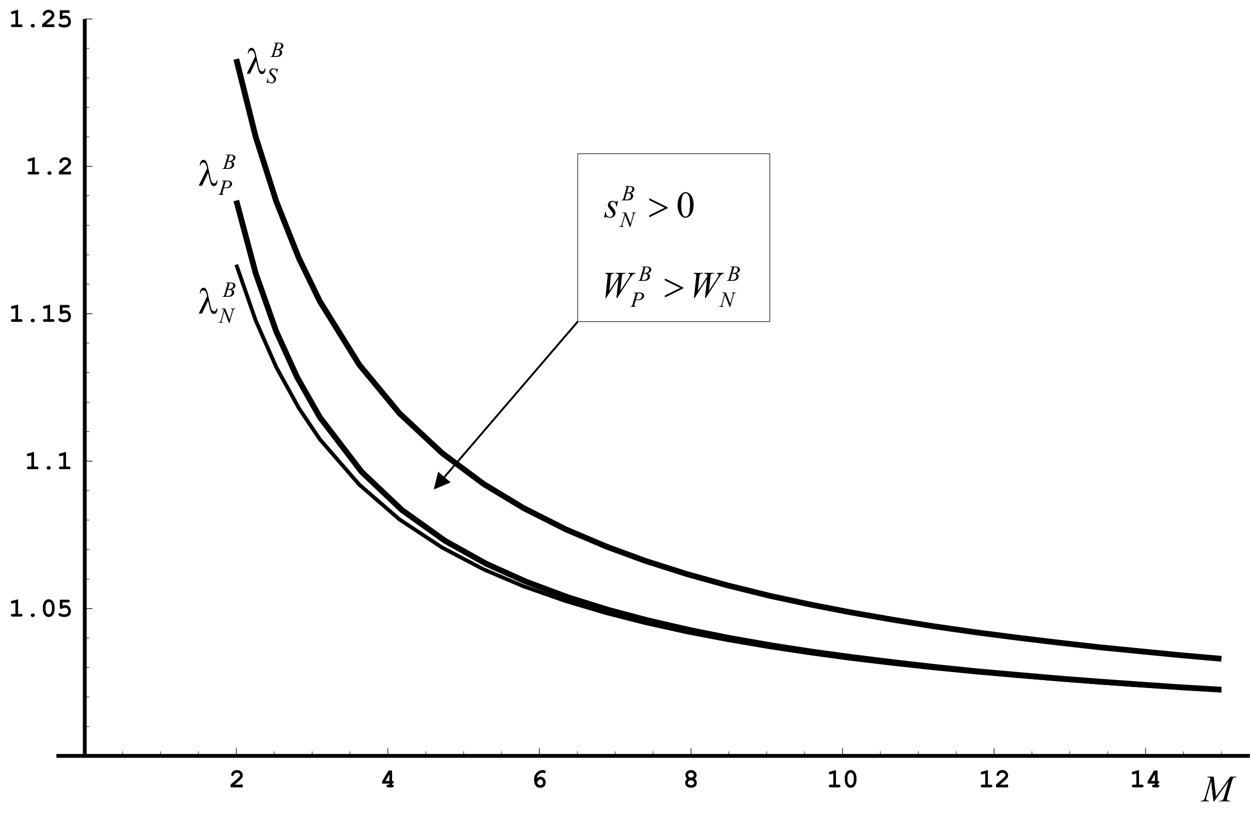


Figure 7: Critical Values of Opportunity Cost for γ/β=3/4 under Bertrand Oligopoly

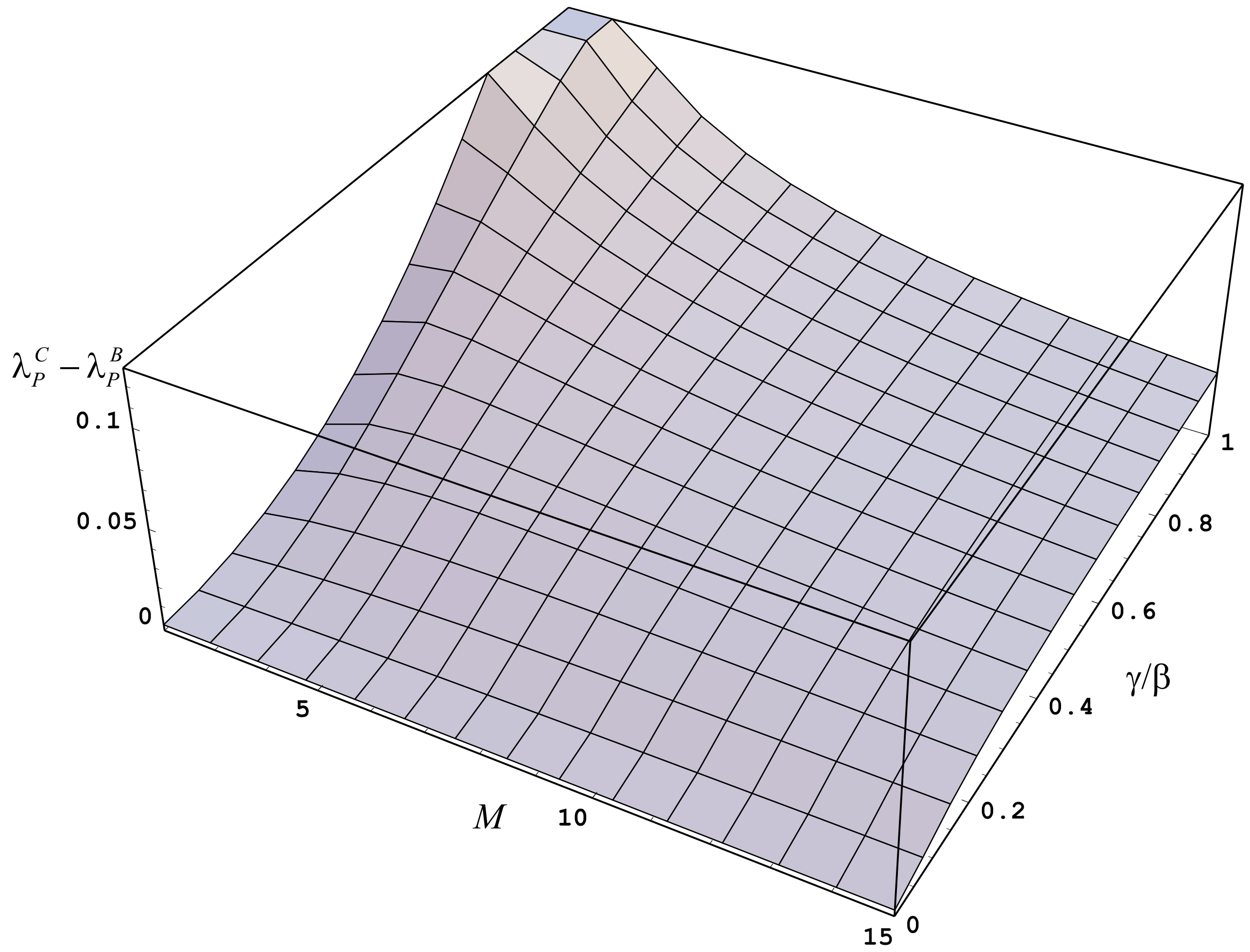


Figure 8: Comparison of Critical Values of Opportunity Cost
under Cournot and Bertrand Oligopoly